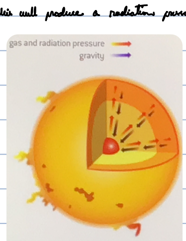
- **Equations:**
  - Parsec definition: $d\ (\text{parsec}) = \frac{1}{p\ (\text{arc-second})}$
  - Luminosity equation: $L = \sigma A T^4$
  - Apparent brightness equation: $b = \frac{L}{4\pi d^2}$
- **Objects that make up the universe**
  - **The solar system**
    - The solar system is a collection of planets, moons, asteroids, comets, and other rocky objects travelling in elliptical orbits around the sun due to its gravitational pull.
    - The sun is thought to have been created from a giant cloud of molecular hydrogen gas that gravitated together.
      - These clouds formed clumps of matter which collapsed and heated up.
      - A gas disc around the sun evolved into the planets.
        - It's thought that planets were formed about $4.6 \cdot 10^9$ years ago.
    - Due to the temperatures being higher closer to the sun, only compounds with high condensation temperatures were left solid.
      - The solid compounds gradually started accreting (sticking together), resulting in the formation of the 4 rock planets, Mercury, Venus, Earth, Mars.
    - As you move further away and the temperature isn't as high, gas giants Jupiter, Saturn, Uranus, and Neptune are located.
      - These planets were formed from cores of rock, metals, and an abundance of ice.
      - Due to the vast presence of ice, the planets became very large, meaning that they produced a strong gravitational field which captured the slow moving helium & hydrogen.
    - The planets form an elliptical orbit around the sun.
      - Only Mercury will occupy a significantly different orbit than other planets.
    - Beyond Neptune lies the Kuiper belt.
    - Most planets have moons.
      - Mars = 2
      - Jupiter = min 50 moons, with several hundred more which are thought to be captured asteroids.
      - Earth = 1
        - Earth's moon is thought to have been created when a Mars sized object collided with Earth.
    - Asteroids are rocky objects orbiting the sun.
      - Asteroids of less than 300 km have an irregular shape because their gravity is too weak to compress them into spheres.
      - Asteroids such as Ceres have a diameter of 10^6 m which make them "minor planets".
    - Comets are irregular objects.
      - They're a few kilometers across and consist of frozen gases, rock, and dust.
      - Observable comets travel around the sun in sharply elliptical orbits with periods ranging from a few years to thousands of years.
      - As the comets move closer to the sun, they will start vapouring (as they're usually made of frozen gases, dust, and rock).
        - This vapouration will result in a tail forming at the back of the comet.
        - The tail can be millions of kilometers long, and always points away from the sun.
- **Stars**
  - Like the sun, all stars are created when gravity causes the gas in a nebula to condense.
    - Nebula: A cloud of gas and dust in outer space.
    - As the atoms move towards one another, they lose gravitational potential energy and gain kinetic energy.
      - This will raise the temperature of the atoms forming a protostar.
        - Protostar: A contracting mass of gas which is the early stages of a star.
        - When the mass of the protostar is large enough, the temperature and pressure at the center of the protostar will be high enough for fusion of hydrogen to occur.
          - The star is said to have been "ignited".
  - Ignition of a star produces emission of radiation from the core.
    - This will produce a radiation pressure which opposes the inward gravitational force.
    -



▲ Figure 4 Hydrostatic equilibrium.

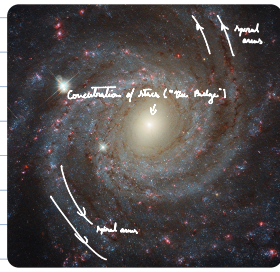
- When the gravity and the radiation pressure equal one another the star is in a state of hydrostatic equilibrium.
  - It will remain stable for billions of years because it's on "main sequence".
  - As the hydrogen is used up, the star will undergo changes.
    - The color of the star will alter as the surface temp will either rise or fall.
    - It will change size.
    - The original mass of the star will determine the changes.

- Groups of Stars
  - It's thought that 50% of the stars nearest to the Sun are part of a system comprising of two or more stars.
    - Binary stars consist of two stars that rotate about a common center of mass. Note: to determine if a system is a binary star or not is to see its
    - A stellar cluster is a group of stars that are close enough to be held together by gravity.
      - Some clusters have only a few dozen stars while others have millions.
      - All stars in the cluster were formed from the same Nebula at the same time.
      - 

        Figure 5 The Pleiades.
      - The image shows a stellar cluster of 500 stars.
        - It is an example of an "open cluster".
          - An open cluster is: a relatively loose grouping of stars.
        - Open clusters consist of up to several hundred stars that are younger than 10 billion years, and may still contain some gas and dust.
    - Globular clusters contain many stars which are older than eleven billion years and, therefore, contain very little gas and dust.
      - ~150 known Globular clusters outside of the milky way.
      - Spherically shaped.
    - A constellation is a pattern formed by stars that are in the same general direction when viewed from Earth.
      - Such stars aren't held by gravity.

- Nebulae
  - Regions of intergalactic cloud of dust and gas are called "nebulae" (singular nebula).
    - All stars are born from Nebulae.
  - There are two different origins for a Nebula.
    - One origin being that it occurred in the "matter era" around 380,000 years after the Big Bang.
      - Dust and gas clouds where formed when nuclei captured electrons electrostatically and produced the hydrogen atoms which gravitated together.
    - The second origin of nebulae is from the matter that is ejected when a supernova explodes.
      - An example would be the Crab Nebula.
      - They can form in the final, red giant, stage of a low mass star such as the Sun.

- Galaxies
  - It's a creation of stars, gas, and dust held together by gravity and containing billions of stars.
    - The Milky Way contains ≈ 3·10¹¹ stars, and, probably, the same amount of planets.
  - Some galaxies exist in isolation, but the majority exist in a cluster.
    - The Milky Way is part of the "Local Group", which includes Andromeda and Triangulum.
    - Regular clusters will contain a concentrated core, and a spherical shape.
      └ Meaning that the galaxies are concentrated in the middle.
    - Irregular clusters are defined as having no apparent shape, and a lower concentration of galaxies.
    - Superclusters also exist.
      - It's a large group of smaller galaxy clusters.
      - Approximately 90% of galaxies are found in superclusters.
  - Spiral galaxies are the most common class of galaxies.
    - They consist of a flat, rotating disk containing stars, gas and dust. With a central concentration of stars known as the Bulge.
      - Internet definition
    - They have a disc-shape with spiral arms spreading out from a central galactic bulge that contains the greatest density of stars.

- Book definition.
- The spiral arms refer to how the galaxy seems to have "arms" stretching out in a circular fashion.
- 

  - It's speculated that at the center is a black hole.
  - The spiral arms contain young stars with young blue stars and a lot of dust and gas.
- Other galaxies are elliptical in shape, being ovoid or spherical.
  - These have much less dust and gas.
  - They're thought to have been created when two spiral galaxies collide with one another.
- Irregular Galaxies.
  - Irregular galaxies have no apparent shape because they have been stretched out by massive galaxies.
    - E.g. the Milky Way is having this effect on smaller galaxies.
- Summing up:
  - There are 3 main clusters (groups) which galaxies exist in.
    - Regular clusters.
      - These are galaxies which have an apparent shape (spherical), and will have a high concentration of stars at the center.
    - Irregular clusters.
      - These are galaxies which have no apparent shape (due to them being stretched out by larger galaxies).
      - They have a low concentration of galaxies in them.
    - Superclusters.
      - A supercluster is a large group made up of smaller clusters.
      - ≈ 90% of galaxies are found in superclusters.
  - Different classes of galaxies exist as well.
    - Spiral Galaxies.
      - These are galaxies such as the Milky Way.
      - They're the most common type of galaxy.
      - High density of stars in the bulge (center of the galaxy).
        - Theory that there is a black hole at the center.
      - The spiral arms contain young blue stars.
        - They will also have a lot of gas and dust.
    - Irregular galaxies are instead irregular in shape.
      - Their irregularity is due to being stretched out by much larger galaxies.
        - I.e. what the Milky Way does to smaller ("dwarf") galaxies.
    - Other galaxies are instead galaxies which are elliptical in shape.
      - They're thought to be created by two spiral galaxies colliding with one another.
      - They contain a lower concentration of dust and gas than spiral galaxies.

- Astronomical distances
  - The light year (ly):
    - $1\ ly = 9.46 \cdot 10^{15}\ m$
    - It takes light from the sun ≈ 7-8 minutes to reach Earth.
  - The astronomical unit (AU):
    - This is the average distance from the sun to the Earth.
    - $1\ AU = 1.5 \cdot 10^{11}\ m \approx 8$ light minutes
  - The parsec (pc) (most common unit in astrophysics):

- $1 pc = 3.26 ly = 3.09 \cdot 10^{16} m$
- Used when measuring the distance between nearby stars is measured in pc.
    - Distant stars within a galaxy = kilo pc (kpc)
    - Distance between galaxies is in mega parsec (Mpc) or gigaparsec (Gpc).

Stellar parallax

- When measuring the distance of a star to planet Earth in Parsec, a similar techniques to finding the distance between two objects is used.
- **Parallax is based on** the fact that **nearby objects** will **appear to cross distant objects at different positions.**
    - As the Earth orbits the Sun, the stars that are quite close to us appear to move past distant ("fixed") stars.
    -

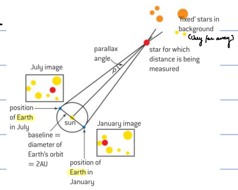


Figure 8 Stellar parallax.

- As seen the star closer to the Earth will appear to be moving in front of other stars as the Earth rotates around the Sun.
    - Given by: $d = \frac{1}{p}$
        - **Distance (d)** is given in parsec (pc), where **p** is the **parallax angle** in arcsecond.
        - This **relationship** is **used** for defining the **parallax angle in arcsecond.**
            - When a star is at a distance of 1 pc from the Earth the parallax angle given by the equation will be one arcsecond.
        - There are 60 **arcminutes in one degree** and **60 arcseconds** in **every arcminute.**
            - Therefore, **1 arcsecond** is $\frac{1}{3600}$ **degrees.**
        - The **limit** for using the stellar parallax is for angles less than **0.01 arcsecond.**
            - This is **due to the fact that** there will be **absorption and scattering of light by the Earth's atmosphere.**
            - **Turbulence** in the **atmosphere** also limits the **resolution** because it causes a **"Twinkle".**
        - With the **parallax equation**, the **maximum range of d** is given by: $d = \frac{1}{0.01} = 100 pc.$
        - When a Telescope is used outside of the atmosphere (i.e Hipparcos), the max distance is increased vastly.

Luminosity and apparent brightness of stars

- The intensity of a wave at a point distance r from the source is given by:
    - $I = \frac{P}{4\pi r^2}$
    - It was **defined** as: the power emitted by a source divided by the area of the sphere over which the energy is spread equally.
        - This essentially means that the intensity is the power divided by the area where the light goes to. Since the sun is a sphere it will emit light in all directions (spherical area).
- For stars, P (Power), is called **luminosity (L)**, and represents the total energy emitted by the star per second in watt.
    - **Intensity** (I) is **equal** to **apparent brightness (b)** ($W m^{-2}$).
    - **Distance** is given by **d**.
    - $I = \frac{P}{4\pi r^2} \longrightarrow b = \frac{L}{4\pi d^2}$

Worked example

- i) **Luminosity**: the luminosity is the total energy emitted by the star per second ($W m^{-2}$).
  ii) **Apparent Brightness**: this is incident power per unit area received at the Earth.
- $b = \frac{L}{4\pi d^2}$ parsec $= 3.09 \cdot 10^{16} m$
  $1.9 \cdot 10^{-9} = \frac{6 \cdot 10^{31}}{4\pi d^2}$ $= 1.63 \cdot 10^{3} pc$ ✓
  $d = 5.01 \cdot 10^{19} m$
- i) The method uses the idea that at closer distances stars will seem to move as the planet goes around the sun.
    - Using the formula $d = \frac{1}{p}$, we can find the distance of the star as long as the parallax angle ≥ 0.01.
  ii) $d = \frac{1}{5 \cdot 10^{-3}}$
  $= 200 pc$

Black-body radiation and stars

- Because there is no reflection or re-emission for black bodies they will appear black.
    - They would also be perfect emitters of radiation, emitting the max amount of radiation at their temp.
- All objects above $\approx 0K$ will have black body radiation.
    - The emissions aren't limited to a single wave length, but instead all wavelengths are emitted for a black body.

- The difference is that the wavelengths emitted will have different intensities.

- Although stars are grey bodies they will be able to absorb all wavelengths of light.

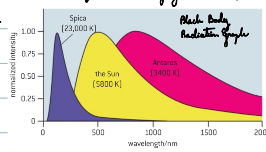
- 

Figure 10 Black-body radiation curves for three stars.

- The diagram shows that the Sun will have a peak λ of ≈ 500 nm, meaning that it will look yellow.
    - The star "Spica", will have a peak in the UV region, but enough intensity of light from the
    - Antares will look red.

- For a star the Stefan-Boltzmann equation is: $L = \sigma A T^4$
    - Where L is the luminosity of the star, measured in watt.
    - A is the surface area of the star.
    - T is the absolute temperature.
    - σ is the Stefan-Boltzmann constant, $5.67 \cdot 10^{-8}$ Wm^{-2} K^{-4}.
    - Assuming that the star is spherical, the equation will be: $L = \sigma A T^4 \longrightarrow L = \sigma 4 \pi R^2 T^4$
        - R = radius of star.

- Worked example
    - 138 pc
      $= 4.26 \cdot 10^{18}$ m
      $= 4.3 \cdot 10^{19}$ m ✓
    - $b = \frac{L}{4 \pi d^2}$
      $L = 3.7 \cdot 10^{31}$
      $= 9.7 \cdot 10^{4}\ L_{sun}$ ✓

- D.2 Stellar characteristics and stellar evolution
    - Equations:
        - Wien's law: $\lambda_{max} T = 2.9 \cdot 10^{-3}$ m K
        - Luminosity-mass relationship: $L \propto M^{3.5}$
    - Stellar spectra
        - When the absorption spectrum of stars are observed a continuous spectrum will be shown with dark absorption lines.
            - The absorption spectrum is a spectrum of electromagnetic radiation transmitted through a substance, where the dark lines are the wavelengths which are absorbed.
                - The absorption lines (dark lines) show that the star has a hot dense region (which produces the continuous spectrum) surrounded by a cooler, low density gas (to produce the absorption lines).
            - In general, the density and the temperature are lower further away from the center.
            - Since the temperature is so high, a star's core has to be composed of high-pressure gases and not of molten rock (unlike some planets).
            - Some stars also show bright emission lines.
                - Emission lines are a spectrum of the electromagnetic radiation emitted by a source.
        - Composition of stars
            - Absorption of certain wavelengths is also shown with the intensity-wavelength relationship for stars.
                - The smooth theoretical black-body curve is modified by absorption dips.

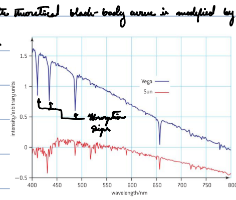


Figure 1 Intensity–wavelength relation for Vega and the Sun.

- The Vega star has the cooler hydrogen in the sun's outer layers (the photosphere) absorbs the photons emitted by the hydrogen.
    - Does this mean that hydrogen on the outside layer of the star absorbing their own photons or those of hydrogen in the sun?
- The absorption of the light in the visible light of the hydrogen absorption spectrum suggest that the photosphere of Vega is almost all hydrogen.

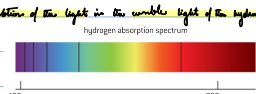
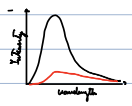


Figure 2 Atomic hydrogen absorption spectrum.

- This shows that the Vega star is part of the Balmer series (visible light series for hydrogen), which equates to n=2 to higher energy levels.
    - Why are there only 4 drops for Vega rather than 5?
- The Sun being a cooler star will have more hydrogen atoms in the ground state (as they're not as excited as they are in Vega).
    - This will mean that they'll be part of the Lyman series.
    - They will produce UV lights.
- Hotter stars than Vega will not produce hydrogen absorption lines in the visible light spectrum.
    - This is due to the hydrogen in the photosphere is ionised, meaning it won't have an electron to become excited by the absorption of a photon.
- Using the absorption spectrum to determine elements in a particular star is problematic as different elements will superimpose on one another.
    - Although, the lines tell us about the temperature.
    - The movement of the gas atoms in the star causes the photons of light emitted to undergo red & blue shifts.
        - This will cause the doppler broadening to be more pronounced.
- The rotation of the star will mean that the light reaching us will come from different parts of the star.
    - One edge moving towards us, the other away, and the center remaining stationary.
        - This increases thermal doppler broadening. — Explain!!

- Wien's displacement and star temperature
    - The temperature, T, of a star in kelvin is given by:
        - $T = \frac{2.9 \cdot 10^{-3} \text{ m K}}{\lambda_{max}}$
            - T is in kelvin, λ_{max} is the maximum intensity of the black-body.

- Worked example
    - The radiation of a black-body is essentially a body which is a perfect emitter of radiation. Meaning that all wavelengths will be emitted indiscriminately.
    - x-axis = wavelength (λ)
    -

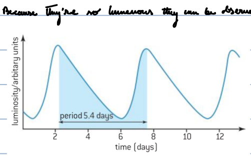


- $\lambda_{max} = 0.97 \mu m$

$T = \frac{2.9 \cdot 10^{-3}}{0.97 \cdot 10^{-6}}$

$\approx 3000K$

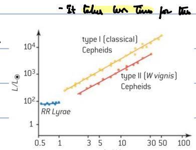
- Cepheid variables
    - Cepheid variables are extremely luminous stars that undergo regular and predictable changes in luminosity.
    - Because they're so luminous they can be observed from the Earth.
    -



▲ Figure 3 Luminosity–time relationship for Delta Cephei.

E.g. Supergiant star, Delta Cephei.

- The period of the Cepheid will vary from 12 hours to a hundred days.
    - Although, the period is regular it's not sinusoidal.
    - It takes less time for the star to brighten than it does to fade.
-



▲ Figure 4 Relative luminosity – period relationships for Cepheid stars.

- The relative luminosity is the ratio of the luminosity of the star to that of the sun.
    - The sun's luminosity is written as $L_{\odot}$.
- Cepheid stars are stars which have completed the hydrogen burning phase and moved on to the main sequence.
    - The variations in luminosity occur due to the outer layer of the star (photosphere) expanding and contracting periodically.
        - Seen here:

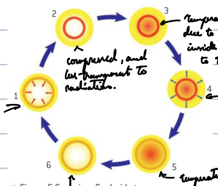


Figure 5 Cycle in a Cepheid star.

- If a layer of an element loses hydrostatic equilibrium (the equilibrium of the gravity and radiation and pressure) then gravity will start pulling it in.
    - this will lead to the layer becoming more compressed and less transparent to radiation.
    - the compression will mean that the temperature inside the layer will increase, leading to pressure buildup.
    - the increase in internal pressure due to the temperature increase will cause the layer to expand.
    - During the expansion of the layer, the temperature will decrease becoming less dense.
    - the lower density will mean that the layer will be more transparent to radiation, this will mean that as the radiation inside escapes, the pressure decreases.
    - this will then mean that the layer will fall inwards due to gravity, repeating the cycle.
        - this constant repeating is what causes the star's radiation to pulse.
- Cepheid variable stars are known as "standard candles", because they let us calculate the distances of how far the galaxies with the Cepheid stars are.
    - this is done with the following equation:
        - $b = \frac{L}{4\pi d^2}$ (b = apparent brightness, L = luminosity), where d is the distance of the Cepheid star.
    - the luminosity is calculated with the period of the Cepheid.
    - the apparent brightness is calculated with a CCD.

Note: Standard candles are defined as: a class of astrophysical objects, e.g. supernovae or variable stars (Cepheid stars), which have known luminosity due some characteristic quality possessed by the entire class of objects.

- Worked example
    - Define: luminosity.
        - luminosity is the total power radiated by the star.
    - Define: apparent brightness.
        - that is the apparent power that we view on planet Earth.
    - c) i) the radius of the Cepheid will be larger after two days because after 6 the star will have been compressed by gravity.
    - c) ii) Cepheid variables are important to estimating distances because with the equation $b = \frac{L}{4\pi d^2}$ we can determine the distance of the galaxy with the Cepheid star.
    - d) i) $b = \frac{L}{4\pi d^2}$

$$d^2 = \frac{L}{4\pi b}$$

$$= \sqrt{\frac{7.2 \cdot 10^{29}}{4\pi (1.25 \cdot 10^{-10})}}$$

$$d = 2.14 \cdot 10^{19}\ m$$

    - d) ii) Standard candles means that they're stars which are used to calculate the distances of the galaxies that they're in.

- Hertzsprung-Russell (HR) diagram
    - Luminosity of a star is proportional to its $Temp^4$ & $radius^2$.
    - Scattergrams of the luminosity - star temp are known as Hertzsprung-Russell (HR) diagrams.
        - In general, the cooler red stars tend to have relatively low luminosity.
        - In general, hot blue stars tend to have high luminosity.
    - On the scattergram, hotter temperatures are on the left of the horizontal axis.
        - the majority of stars will create a band from top left to bottom right, known as the main sequence.
            - Some stars aren't part of the main sequence, and exist as "islands" above and below the main sequence.
        - the y-axis is to show the luminosity of the star in comparison to the sun ($L_\odot$).
    - During the lifetime of the star its position on the HR scatterplot will change.
    - Small, cool stars will be at the bottom right of the HR diagram as luminosity is given by $L = \sigma A T^4$.
        - According to Wien's law (graph of temp - λ) the stars are going to be red.
    - Large, hot stars will be at the top left of the graph, and will be blue/blue-white.

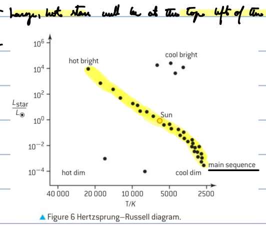


Figure 6 Hertzsprung–Russell diagram.

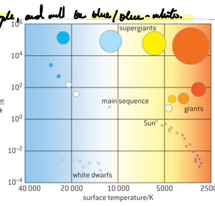


Figure 7 Hertzsprung–Russell diagram showing the different classes of stars.

- Main sequence stars are ordinary stars, such as the sun, that produce energy from fusion of hydrogen / helium / carbon.
    - ≈ 90% main sequence.
- Different kinds of stars and their luminosities:
    - Red Giants.
        - these stars are cooler than the sun, meaning that they emit less energy per square meter of surface area.
        - they have a higher luminosity than the sun, emitting up to 100 times more energy than the sun.
            - this means that the surface area is huge to emit such large energies.
                - this makes them "giants".
    - Supergiant Stars
        - they are gigantic and very bright.
            - Up to 100,000 times the energy per second at the same temp. of the sun.
            - Up to 100,000 times the surface area of the sun.
                - Over 300 times the diameter of the sun.
        - Only 1% of stars are giants & supergiants.
    - White dwarfs
        - these are remnants of old stars.
        - ≈ 9% of all stars = white dwarfs.
        - Although they're very hot, they have a low luminosity because they're so small (low surface area).
        - they take billions of years to cool off.
- Mass - luminosity relation for main sequence stars
    - Not all main sequence stars are like the sun, some are hotter and larger, while some are smaller and cooler.
    - High mass stars have shorter life spans.
        - A star with 10 times the solar mass (mass of the sun) might only have a lifespan of 10 million years.
        - The sun on the other hand has an ≈ lifespan of 10 billion years.
    - The relationship between the luminosity and the mass is:
        - $L \propto m^{3.5}$
        - E.g. a star with 10 times the mass of the sun will have a luminosity $10^{3.5} \approx 3200$ times greater than the sun.
    - For a star to be stable it has to be in hydrostatic equilibrium.
        - Hydrostatic equilibrium is the pressure inwards the star due to gravitational attraction of inner shells is equal to the outwards force due to thermal and radiation pressure.
        - For larger stars, the gravitational force inwards will be greater, therefore, the core temp. will be higher.
            - The higher temperature will mean that fusion between nuclei is more probable giving a greater rate of nuclear reaction and emission of more energy.
                - this will result in a higher luminosity.
- Stellar evolution
    - Formation of a star
        - The initial formation of a star is due to the gravitational attraction of hydrogen nuclei.
        - The loss of potential energy leads to an increase in gas temp.
            - The gas becomes denser, and when the protostar (initial phase of star) has sufficient mass, the temp. will be high enough for fusion to occur.
        - The star will then move onto the main sequence where it remains as long as it has the hydrogen to fuse into helium.
            - Eventually, when most of the hydrogen in the core has fused into helium the star moves off the main sequence.
    - The fate of stars
        - All stars collapse when most of the hydrogen nuclei have fused into helium.
            - Gravity will out weigh the radiation pressure and the star shrinks in size and heats up.
            - The hydrogen in the layer surrounding the shrunken core is able to fuse, raising the temp. of the outer layer making them expand, forming a giant star.
                - The fusion of hydrogen adds more helium to the core which continues to shrink and heat up, forming heavier elements such as Carbon and Oxygen.
        - Super massive stars will be able to continue undergoing fusion until nickle and iron (the most stable elements) are formed.
    - Sun-like Stars
        - For stars like the sun of moderate mass (up to 4 solar masses), the core temp. won't be high enough to allow fusion of carbon.
            - Meaning that when Helium is used up, the core will continue to shrink and radiate.
                - this will "blow away" outer layers forming a planetary nebula around the star.
        - When the remnant of the core has shrunk to about the size of the Earth it consists of Oxygen and Carbon surrounded by free electrons.

- The core is prevented to shrink any further by an electron degeneracy pressure.
    - Pauli's exclusion principle prevents two electrons from being in the same quantum state, meaning gravity won't be able to further collapse the star as electrons will provide a repulsive force against one another.
- The star is left to cool for billions of years as a white dwarf.
    - These stars are high density, c 10^{9} kg m^{-3}.
    - Sirius B was the first white dwarf to be identified.
- Probable future of the Sun.
    -

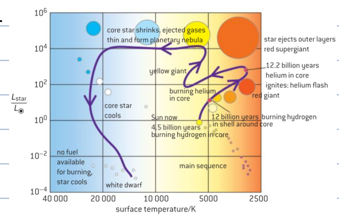


Figure 9 Hertzsprung–Russell diagram showing the Sun's path.

- Larger Stars
    - Stars with a larger core than the Sun's will undergo a different evolutionary path from Sun-like stars.
    - When such large stars are in the red giant phase, their core will be so large that they will be able to form elements which are heavier than Carbon, through fusion.
        - The red giant phase ends with the stars having layers of elements with decreasing proton numbers from the core to the outside.
        - The dense core causes gravitational contraction (the inward collapse of a star due to its own gravity).
            - This is opposed for lighter stars by electron degeneracy pressure.
            - Even with electron degeneracy pressure larger stars can't stabilize.
    - The Chandrasekhar limit states that it's impossible for a white dwarf to have a mass of more than 1.4 times the mass of the Sun. Stars that end up with a mass lower than 1.4 solar masses become white dwarfs.
        - When the masses of the white dwarfs reach this value, the electrons combine with the protons to form neutrons, emitting neutrinos in the process.
        - The star collapses with neutrons coming as close to one another as in a nucleus.
        - The outer layers of the core rush in towards the core but bounce off in a huge explosion, a supernova.
            - This blows off the outer layer forming a neutron star.
    - Similarly to electron degeneracy, neutrons will provide neutron degeneracy pressure preventing further collapsing of the star.
        - The Oppenheimer-Volkoff limit states that the neutron degeneracy pressure can only withstand further collapse into a black hole if the neutron star is between 1.5–3 solar masses.
    -

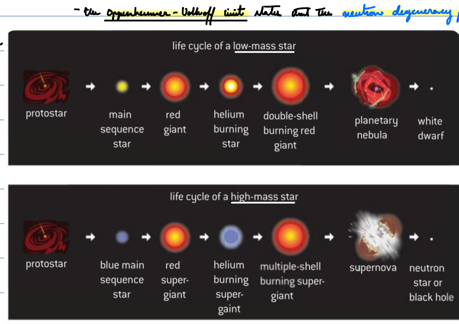
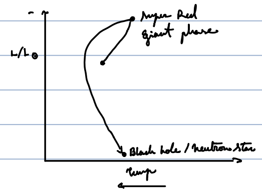


Figure 11 The evolution of Sun-like and more massive stars.

- Black holes
    - It's not possible to form a neutron star having a mass greater than the Oppenheimer-Volkoff limit (1.5–3 times solar masses).
        - Instead, the remnant of a supernova will form a black hole.
    - Nothing can escape a black hole.
    - Their existence is proven by the following:
        - The X-Rays emitted by matter spiralling towards the edge of a black hole and heating up.
        - Giant jets of matter have been seen to have been emitted by the cores of some galaxies.
            - It's suggested that only black holes are powerful enough to produce such jets.
        - Unimaginably strong gravitational fields have been seen to influence stars in the vicinity, causing them to spiral.
            - A black hole has been detected in the centre of the Milky Way, and it's suggested that they're at the centre of all galaxies.
- Worked example
    - The condition is that the Sun will have had to finish the majority of its hydrogen fuel, leading to the core becoming smaller and hotter meaning that the layers around the core would be able to undergo

fusion, causing the outer layers to expande due to the high temperatures.

- The luminosity will increase as the surface area of the sun will increase ($L = \sigma A T^4$), this is due to the sun increasing in size due to the fusion occuring on the outer layer of the core.
- The reason that the sun will end up as a white dwarf is because during its Red Giant phase, it will have a mass lower than 1.4 solar masses, resulting in it becoming a white dwarf.
- The star will go into a super giant red star, and since it will have a mass which is greater than 1.4 solar masses, it will end up becoming either a black hole or a neutron star.
  - It'll be a black hole if its Neutron Star will be greater than 3 solar masses.



- D.3 Cosmology
  - Equations
    - The redshift equation: $z = \frac{\Delta\lambda}{\lambda_0} \approx \frac{v}{c}$
    - Relation between redshift and cosmic scale factor: $z = \frac{R}{R_0} - 1$
    - Hubble's law: $v = H_0 d$
    - Age of the universe estimate: $T \approx \frac{1}{H_0}$
  - The redshift and Hubble's law
    - In the 1920s, Hubble compared the spectra from distant galaxies to Earth bound ones, and discovered that galaxies where redshifting in line with the Doppler effect.
      - This means that galaxies are moving away from us.
      -

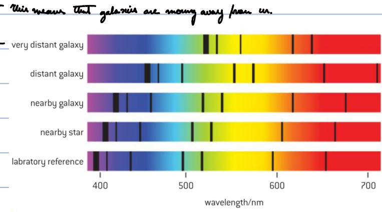


Figure 1 Redshifted absorption spectra.

- For optical spectra the wavelengths are moved towards the red end of the spectrum.
  - The shift applies to all wavelengths in the spectrum, so the absorption lines in the spectrum can be seen to have shifted.
- The further away the galaxy is, the greater the redshift.
- Hubble's law suggested that the recessional speed of the galaxy is proportional to the distance from Earth.
  - Hubble's law: $v = H_0 d$
    - v = velocity of recession, d is the distance of the galaxy (both measured from Earth), and H_0 is Hubble's constant.
    - $H_0 \approx 70\ km s^{-1} Mpc^{-1}$.
- Worked example
  - The further away you are from a galaxy, the harder it is to accurately determine the velocity.
  - It would give us the precise distance of the galaxies from Earth.
  - $v = H_0 d$, $d = \frac{v}{H_0} = \frac{4.1 \cdot 10^7}{70}$, $d = 590\ Mpc$
    $\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$, $v = \frac{90 \cdot 10^{-9} \cdot 3 \cdot 10^8}{660 \cdot 10^{-9}}$, $v = 4.1 \cdot 10^7 ms^{-1}$
- The Big Bang model and the age of the universe
  - Hubble proving the galaxies are moving further away proves that at one time they were much closer.
    - According to the standard model, 13.7 billion years ago the universe occupied a space smaller than the size of the atom.
    - At the instant that the entire universe exploded in a Big Bang, undergoing immense expansion where both time and space came to be.
  - The universe started at 10^{32} K, it rapidly cooled to 10^{10} K after one second.
    - In the time since the Big Bang the universe has cooled to 2.7 K.
  - In this time there has been an expansion of the fabric of space.
    - As the galaxies move further apart, the space already between them becomes stretched.
      - This is what is meant by expansion, and why redshift occurs.
      - The space through which the electromagnetic radiation travels is expanding and it stretches out of wavelength of the light.
        - The further away the source of the light, the greater the space becomes stretched, resulting in more stretched-out wavelengths, and meaning the

cosmological redshift.

- Hubble's law can be used to estimate the age of the universe.
  - Assuming that Hubble's law has held true for all galaxies at all time, the light from the most distant star has taken the age of the universe to travel to us.
  - If the light was emitted immediately after the big bang, the space between the galaxy & Earth must have expanded at a speed slightly less than the speed of light for the light to reach us.
    - this makes the recessional speed of the galaxy $\simeq c$.
    - the distance that the light has travelled from the galaxy = $c \cdot T$, where T is the age of the universe.
    - $v = H_0 d$
      $c \simeq H_0 c T$
      $T \simeq \frac{1}{H_0}$
      - Using $H_0 = 70\,km\,s^{-1}\,Mpc^{-1}$ ($1\,Mpc = 3.1 \cdot 10^{22}\,m$)
      - $\frac{1}{H_0} = \frac{3.1 \cdot 10^{22}}{70 \cdot 10^3} = 4.4 \cdot 10^{17}\,s$
        $= 1.4 \cdot 10^{10}\,yrs$ (14 billion years).
      - the duration of the universe assumes that the galaxy and the Earth are moving at a relative constant speed of c.

- The importance of the cosmic microwave background (CMB)
  - In the 60s there were two theories for the universe's beginning, the Big Bang, and the steady state theory.
  - It was predicted that the universe should show the spectrum of a black-body emitter at a temp of about 3k.
  - In the Big Bang model, at approx $4 \cdot 10^5$ years after the Big Bang, the temp had cooled to 3000k and the charged ion matter was able to attract electrons to form neutral atoms.
    - this meant that space had become transparent to electromagnetic radiation, allowing it to escape in all directions.
    - the expansion of the universe meant that photons emitted at this time has been shifted to peak at 7cm, while at earlier stages of the universe it would peak at the UV and visible light spectrum.
      - Again to reiterate, expansion means that the space between the galaxies is being stretched out.
  - The CMB in the sky looks essentially the same in all directions (isotropic).
    - It doesn't vary with the time of day.
  - Essentially
    - CMB is the remenant radiation from the early stages of the universe.
      - the radiation fills all space between galaxies.
      - the glow is strongest in the microwave region.
        - As the universe expanded, the wavelengths have gotten longer.
      - What CMB is photons that originally propagated at the start of the universe, and as the universe expanded, they filled up all of space.
        - these can be seen because when the electrons and protons merged together to make neutral atoms, the radiation was able to pass through, rather than undergo scattering.
        - As the universe has expanded, the photon's have become less energetic and their wavelengths peak in the microwave region.

- The redshift equation and the cosmic scale factor
  - the redshift ratio is given by: $\frac{\Delta\lambda}{\lambda_0}$, and is denoted by the symbol z, giving:
    - $z = \frac{\Delta\lambda}{\lambda_0} \simeq \frac{v}{c}$
  - Since CMB suggests that the universe is isotropic, and homogeneous at any point in space at a chosen time after the BB, it's essentially true that the density of matter should be the same throughout the universe.
  - As the universe expands, all distances are stretched with the cosmic scale factor, R. The Cosmic Scale Factor represents the relative expansion of the universe.
    - I.e., if radiation had a wavelength of λ_0 when it was emitted, but λ when it was detected. the cosmic scale factor would have changed from R_0 to R.
    - Meaning, space stretched out by ΔR, and the wavelength stretched out by $\Delta\lambda$.
  - $\frac{\Delta\lambda}{\lambda_0} \simeq \frac{v}{c}$ and $z = \frac{\Delta\lambda}{\lambda_0} = \frac{\Delta R}{R_0} = \frac{R - R_0}{R_0} = \frac{R}{R_0} - 1$

- Worked example
  - $z = \frac{\Delta\lambda}{\lambda_0} \simeq \frac{v}{c}$
    $v = 5.6 \cdot 3 \cdot 10^8$
    $\simeq 1.7 \cdot 10^9\,ms^{-1}$
  - $\frac{\Delta\lambda}{\lambda_0} = \frac{R}{R_0} - 1$
    $6.6 = \frac{R}{R_0}$
    $\frac{R_0}{R} = 0.15$
    $= 15\%$ of size now.

- Type of super nova
  - type Ia supernovae is used as a standard candles to calculate galactic distances of up to 1000 Mpc and measuring their redshifts.
    - these show that the universe is undergoing accelerated expansion.
    - since there are objects in the universe which have the tendency to slow down expansion, the acceleration must be due to objects we can't see.
      - this is known as "dark energy".
  - Source of expanding universe:

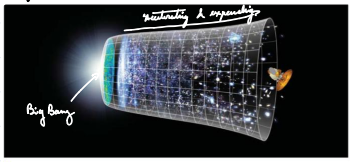
- 

Figure 5 Evolution of the accelerating universe.

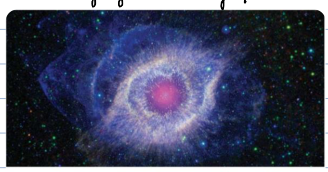
- D.4 Stellar processes HL
  - The Jeans criterion for star formation
    - As you know, stars form out of nebulae, interstellar clouds of dust and gas (hydrogen, helium, and heavier elements (e.g. Helix nebula)).
      - these clouds can exist for millions of years in a relatively constant state.
        - Although, they'll be constantly gaining and losing gas from the regions around them.
        - Eventually, something occurs, e.g. supernova in vicinity exploding or collision with other cloud, which disturbs the calm.
          - this will mean that the cloud will become unstable, and could collapse.
    - Nebula are usually very cold, at times only a few kelvin above absolute zero.
    - 

      Figure 1 The Helix Nebula.

    - Whether or not the gravitational attraction of the gas is sufficient for star formation depends on how quickly the gas temperature rises to prevent the gas from free-falling into the center of the cloud.
      - Compressions in the gas travel at the speed of sound ($340 m s^{-1}$).
      - With small quantities of gas, the compressions pass through it quickly enough to prevent it from collapsing.
        - The gas will oscillate and stabilize.
    - With larger quantities of gas, the compressions travel too slowly to prevent collapse.
    - Jeans' Criterion is what is used to determine if there is sufficient mass for gravity to overcome radiation pressure.
      - The total energy of a gas cloud is a combination of positive kinetic energy, and negative gravitational potential energy.
        - At infinite separation of the gas particles, their entire energy will be kinetic.
          - This is due to the fact that the particles won't be bound to one another.
          - When they're close and the potential energy is dominating, the particles are bound.
            - Total energy is negative.
      - The Jeans Criterion for a gas to collapse is simply, the magnitude of the potential energy must be greater than the kinetic energy.
        - This depends on the temp. and the particle density in the cloud.
        - A cold, dense cloud is far more likely to collapse than a hot low-density gas.
          - The hot low-density gas will have too much kinetic energy, and not enough gravitational potential energy.
          - Usually a cold dense cloud will have $3 \cdot 10^{7}$ particles per cubic meter, and a temperature of around 100K.
            - Under these circumstances, the "Jeans Mass" must be around 400 thousand solar masses to facilitate star formation ($M_J > 400{,}000\ M_\odot$).
            - Individual stars have a mass from $0.1 - 150\ M_\odot$.
        - In cooler regions of space, where the gas clouds have a higher density $\approx 10^{11}$ particles per cubic metre & temps of 10K, the Jeans mass will fall to $50\ M_\odot$.
          - With these masses, short-lived giant stars are formed.
  - Nuclear fusion
    - The energy generated by a star is a result of thermonuclear fusion reaction which takes place in the core of the star.
      - This is known as "hydrogen burning", where hydrogen is turned into Helium.
    - For stars like the sun, the process continues through proton-proton chain reactions.
      - For larger stars, greater than 4 solar masses, they undergo a series of reactions known as CNO cycle.
    - The proton-proton chain reaction has 3 stages:
      - ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{1}e + {}^{0}_{0}\nu$
        - Two protons (hydrogen-1 nuclei) fuse into a hydrogen-2 nucleus (plus a positron and a neutrino).
      - ${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma$

- A hydrogen-2 collides with a hydrogen atom, forming a Helium atom and a gamma ray photon.
- $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$
  - Two helium atoms will fuse to form Helium-4 and two hydrogen-1 nuclei.
- Therefore, in order to produce a helium-4 nuclei, 4 hydrogen are used (6 are actually used, but 2 are produced).

- The CNO process occurs in larger stars with a mass greater than 4 solar masses, and a minimum core temperature of $2 \cdot 10^{7}$ K.
  - Reaction:
    - $^{1}_{1}H + ^{12}_{6}C \rightarrow ^{13}_{7}N + \gamma$
      - Proton (hydrogen-1) fuses with carbon-12 to give unstable nitrogen-13 + a gamma ray photon.
    - $^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{1}e + ^{0}_{0}\nu$
      - Nitrogen-13 undergoes positron decay into Carbon-13
    - $^{1}_{1}H + ^{13}_{6}C \rightarrow ^{14}_{7}N + \gamma$
      - Carbon-13 fuses with proton to give nitrogen-14 + a gamma ray.
    - $^{1}_{1}H + ^{14}_{7}N \rightarrow ^{15}_{8}O + \gamma$
      - Nitrogen-14 fuses with proton to give unstable oxygen-15 + a gamma ray photon.
    - $^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{1}e + ^{0}_{0}\nu$
      - Oxygen-15 undergoes positron decay into Nitrogen-15.
    - $^{1}_{1}H + ^{15}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$
      - Nitrogen-15 fuses with proton to give carbon-12 (again) and helium-4.
  - 4 protons are used to undergo the fusion process, where C-12 is both a fuel and a product.
    - 2 positrons, 2 neutrinos, and 3 gamma rays are emitted.
    - The fusion of hydrogen into helium takes up the majority of the stars life.
      - This is why there are so many stars in their main sequence phase rather than anywhere else.
  - For the CNO cycle while the final step will have another C-12 formed, it will cancel out at the start, meaning only Helium is formed from hydrogen.

- Fusion after the main sequence
  - Once the hydrogen in the core is used up, the core converts to helium.
    - The lack of radiation pressure will cause the core to shrink.
    - This will increase the temperature of the core, meaning that the layer on the core of the stars will start fusing hydrogen.
      - This will mean that more helium is produced, resulting in the core continuing to shrink and produce heavier elements.
      - This will mean that the layers around the outer layer of the core will start expanding, causing the Red Giant Phase.
  - During the Red Giant Phase, the core's temperature becomes high enough for the production of heavier elements, such as: Carbon, Oxygen, Nitrogen, etc.
    - Helium will fuse into unstable beryllium which then fuses with a helium nucleus to produce carbon and then oxygen:
      - $^{4}_{2}He + ^{4}_{2}He \rightarrow ^{8}_{4}Be$
        - Two helium nuclei fuse to produce unstable beryllium-8.
      - $^{4}_{2}He + ^{8}_{4}Be \rightarrow ^{12}_{6}C$
        - A helium nuclei and an unstable Beryllium nuclei will fuse together to produce C-12.
      - $^{4}_{2}He + ^{12}_{6}C \rightarrow ^{16}_{8}O$
        - Another helium nuclei will fuse with C-12 to produce Oxygen-16.
  - These reactions are called "nucleosynthesis" meaning, the production of different nuclides by the fusion of nuclei.
  - Eventually, the C-12 and O-16 are going to undergo fusion to silicone, magnesium, and sodium. Until they reach iron-56.
    - Iron-56 is one of the most stable elements. Although, Nickel-62 is more stable, it's less abundant in stars.
      - These nuclei have the highest binding energy per nucleon.
      - Energy won't be released by further fusion, but rather energy will have to be put in.
  - Since nickel-62 is the most stable element, heavier elements are formed by neutron capture.
    - Since neutrons don't have a charge, they will be able to move in close to a nucleus and be captured by the strong nuclear force.
    - This will increase the nucleon number, therefore, it won't produce a heavier element but rather an isotope.
      - The isotope will undergo gamma ray decay and produce a less energetic sibling.
      - The neutron will either be stable, or it might undergo decay into a proton, an electron, and antineutrino (β^{-} decay).
        - This will increase the proton number, creating a new element.
        - The new element will be excited, and will undergo gamma ray decay to decay into a less energetic nuclide.
    - The half-life of Beta Decay is dependent on the nature of the parent nuclide.

- If there is enough time for further neutron capture will depend on the density of neutrons bombarding the nuclei.

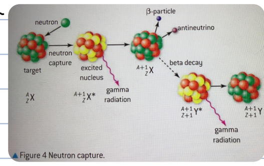
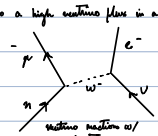
- 

Figure 4 Neutron capture.

- In massive stars, heavy nuclides (up to bismuth-209) can be produced by slow neutron capture (s-process).
  - The stars provide a small neutron flux as a by-product of carbon, oxygen, and silicone burning.
    - This means that there is time for nuclides to undergo beta decay before neutron captures increase the nucleon number.
- The rapid neutron capture, "r-process", there's insufficient time for beta decay to occur.
  - Therefore, heavy elements are built up very quickly, one neutron at a time.
  - Type II supernovae produce a very high neutron flux and form nuclides heavier than bismuth-209 in minutes.
    - Even before there is any likelyhood of beta decay occuring.
    - Type II supernovae are produced by the rapid collapse and violent explosion of a massive star.
      - They must be at least 8 times but no more than 40 to 50 times the solar mass ($M_{\odot}$).
  - r-process doesn't seem to occur in massive stars.
  - There is also a high neutrino flux in a supernova and this has the effect of causing neutrons to convert into protons through the weak interactions, forming new elements.
    - 

      neutrino reactions w/ neutron

      $n + \nu \rightarrow p + e^-$

- Worked example
  - Nickel-63 is the most stable element, meaning that it will have the highest nuclear binding energy per nucleon meaning that energy will have to be put in for the reaction to occur.
    - In chemistry terms, the reaction is endothermic meaning that it won't be spontaneous.
  - The way that heavier elements are produced, is by neutron capture.
    - What this is, is a massive star will form heavy elements by slow neutron capture (s-process).
      - This will lead to the products undergoing Beta minus decay increasing the nucleon number. It can reach a mass of Bismuth-209.
    - For supernovas (specifically II supernova) will undergo rapid neutron capture.
      - This will mean that the products won't be able to undergo β^- decay producing elements heavier than Bismuth-209.
- Lifetime of main sequence stars
  - The more massive the star the shorter its lifespan.
    - Massive stars need a higher core temperature and pressure to prevent gravity from collapsing the star.
    - This means that fusion will occur in the core at a higher rate compared to smaller stars.
      - This will result in the stars finishing their core hydrogen faster than smaller stars.
  - As stated before: $L \propto M^{3.5}$
    - Luminosity is the total energy E released by a star per unit time while H_2 is being fused, or: $L = \frac{E}{t}$
    - While energy is emitted (due to the fusion process), a loss of mass will accompany it.
      - This will amount to "k" during a star's lifetime. Meaning star of mass M will lose a mass of kM.
      - This makes the energy emitted by the star during the hydrogen burning phase: $E = kMc^2$.
    - The average luminosity is given by:
      - $L = \frac{kMc^2}{T}$ (T is the lifetime of the star)
      - Since $L \propto M^{3.5} \rightarrow T \propto \frac{M}{M^{3.5}} \rightarrow T = M^{-2.5}$
  - The sun is expected to have a lifetime of 10^{10} years (10 billion years).
    - A bigger star would mean that it would have a shorter lifespan.
      - E.g. Star mass = $10M_{\odot} \rightarrow \frac{T_{star}}{10^{10}} = \left(\frac{10M_{\odot}}{M_{\odot}}\right)^{-2.5}$
        $= 3.2 \cdot 10^{-3}$
      - This will give a lifetime of $3.2 \cdot 10^{7}$ years
      - This is equal to 0.3% of the sun's life time.

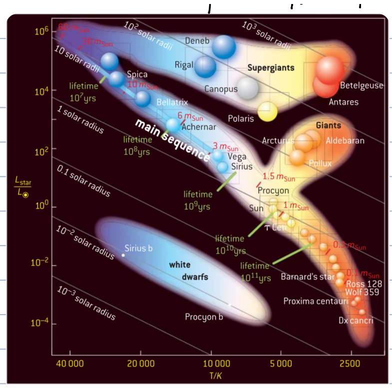


Figure 6 HR diagram.

- Worked example
  - $\frac{T_{star}}{10^{10}} = \left(\frac{30\, M_\odot}{M_\odot}\right)^{-2.5}$ $= 5.66 \cdot 10^5$ years — the reason it has a shorter lifespan is because it will have a core at a higher temp., meaning it will undergo fusion more readily resulting in the Hydrogen being used up faster, and the star collapsing under gravity sooner.
    - Furthermore $T \propto m^{-2.5}$.

- Supernovae
  - Supernovae are very rare events in any given galaxy.
    - They appear as very bright stars in positions that were previously unimpressive in terms of brightness.
  - Supernovae are classified as type I or II depending on their absorption spectra.
    - Type I will have no hydrogen lines as they're formed from old, low-mass stars.
      - Type I stars are further classified as Ia, Ib, Ic. Note: A, B, C,... will be the groups for stars of that temp. Acronym = O, B, A, F, G, K, M
      - This will depend on aspects of their spectra.
    - Type II will have a hydrogen line as they're formed from young, high-mass stars.
  - Type Ia supernovae result from accretions of matter between two stars in a binary star (systems with two stars rotating one common point).
    - This means that the two stars came together to form a supernovae Ia.
    - One of the stars is a white dwarf, and the other is either a smaller white dwarf or a giant star.
    - The formation of these supernovae will result in a rapid increase in brightness, followed by a gradual decrease.
  - Type IIa supernovae are formed by a singular massive star in the final stages of its evolution.
    - They will produce light curves of different characteristics.

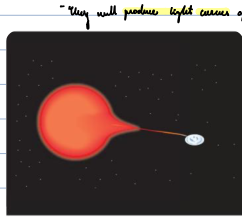


Figure 7 An artist's impression of the accretion of stellar matter leading to a Type Ia supernova. The white dwarf is the star on the right, while the left-hand star is a red giant.

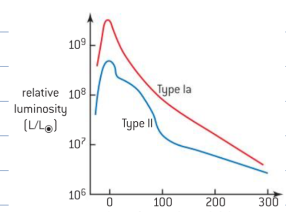


Figure 9 Light curves for class Ia and II supernovae.

- Type Ia Supernovae
  - They will emit light in a certain way and behave as standard candles for measuring the distance of the galaxy that they're in.
  - Given that white dwarfs are extremely dense, they'll attract the matter of the second star.
  - When the mass of the growing white dwarf surpasses Chandrasekhar's limit (1.4 $M_\odot$), the star will collapse under gravity.
    - The fusion of Carbon & Oxygen into Nickel will generate so much radiation pressure that the star will be blown apart, reaching a luminosity of 10^{10} times that of the Sun.
    - Because the chain reaction always occurs at this mass (1.4 $M_\odot$), the brightness of it will be known, and comparing it with the apparent brightness observed on Earth, we can estimate the supernova's galaxy up to 1000 Mpc.
  - After the explosion the ejected materials continue to expand in a shell around the remnant for thousands of years until it mixes with interstellar material.
    - Giving the potential for the next gen of stars.
- Type II Supernovae
  - After ≈ 10 million years, for stars of 8-10 $M_\odot$, all the hydrogen in the core is converted into He.
    - Hydrogen fusion will only continue around the helium core.

- The core undergoes gravitational collapse until its temperature is high enough for fusion of helium into carbon and oxygen.
    - This phase lasts for about a million years until the core is exhausted.
- Once the helium has been consumed, the core will continue to shrink until the temperature is high enough to fuse carbon into heavier elements.
    - It takes about 10 thousand years to exhaust the carbon.
- This pattern continues where a heavier element will last for less time.
    - Until silicon is fused into Iron-56.
- At this point, the star won't be at hydrostatic equilibrium.
    - This is due to the fact that there will be too little radiation pressure to oppose the gravity.
    - Therefore, once the Chandrasekhar limit ($1.4 M_\odot$) is reached, the electron degeneracy won't be strong enough to oppose gravity.
        - This will mean that the star will implode producing neutrons and neutrinos.
        - The implosion is opposed by neutron degeneracy pressure that causes an outward shock wave.
            - This passes through the outer layers causing fusion to occur.
                - The fusion process lasts for only a few hours, but it forms many heavy elements.
            - As the shock wave reaches the edge of the star, the temp. will increase instantly to 20,000 K.
                - This will make the star explode, blowing material off as a supernova.

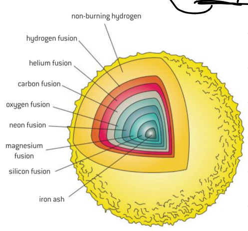


Figure 8 The onion model of a massive star before it goes supernova

- Type Ia and II stars are distinguished by their brightness differences.
    - Type Ia emit up to 10^{10} times the luminosity of the sun.
        - This rapidly reaches a maximum and gradually tails off over 6 months or so.
- Type II emits up to 10^{9} times the luminosity of the sun.
    - It will fall slightly, plateau for a few days, and fall rapidly.

- D.5 Further Cosmology HL
    - Equations:
        - Velocity of rotating galaxies: $v = \sqrt{\frac{4\pi G}{3}\rho}\, r$
        - Critical density of universe: $\rho_c = \frac{3H^2}{8\pi G}$

    - The Cosmology Principle
        - Two assumptions of the universe which are true are:
            - The universe is homogenous.
                - Meaning the same.
            - The universe is isotropic.
                - The same measured value when measured in different directions.
        - These two assumptions are known as the "Cosmological Principle".
            - Defined as: "viewed from sufficiently large enough scale, the properties of the universe are the same for all direction".
        - The image produced by the Automated Plate Measurement (APM) shows that there is no special place or region for the 3 million observed galaxies.
            -

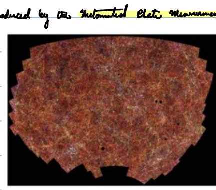


Figure 1 APM Galaxy survey image.

        - With the cosmological principle and the general theory of relativity, it can be seen that matter can only distort spacetime in one of three ways.
            - The flat surface can be positively curved into a spherical shape of a finite size.
                - This will mean that travelling on the surface of the sphere you'll return back to your original position.
                - This will also be the case if you travel through it.
            - The flat surface can be negatively curved into a saddle.
                - Here you'll never return to the same place.
            - The surface can also remain flat and infinite.
                - You'll never return to the same point.
        - There is a critical density (ρ_c) of matter that would keep the universe flat and infinite.
            - This density would provide a gravitational force large enough to prevent

Density parameter: $\Omega_0 = \frac{\rho}{\rho_c}$

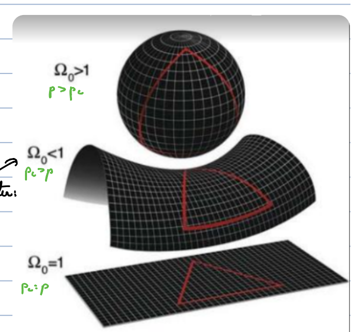


Figure 2 The visualizations of closed, open, and flat universes.

the universe running away, but too small to pull it back to its initial state.

- With less than critical density, the universe would be open and infinite. The saddle in the sphere.
- With a greater critical mass the universe would be closed and finite. The sphere in the universe.
    - Gravity would be pulling matter back to its initial state of singularity.
- The critical density seems to be no greater than 10 particles per cubic meter.
    - Current research shows that the density is very close to this amount.

- The implications of the density of intergalactic matter
    - The theory suggests that the universe has been expanding at an accelerating pace.
        - This is due to matter which we can't detect, dubbed "dark matter".
    - What is essential to know is the amount of mass needed to provide a strong enough gravitational force to reverse the expansion and cause a gravitational collapse.
    - The relationships for critical density can be derived with Newtonian mechanics:
        - Imagine a homogeneous sphere of gas, with radius r and density ρ.
        - A galaxy of mass m will move away with a recessional speed of v away from the center of the sphere along a radius.

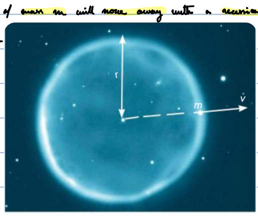


Figure 3 Critical density for the universe.

- Hubble's law is used to find the velocity of the galaxy: $v = H_0 r$
- The total energy of the galaxy is its E_k and GPE relative to the center of the universe/sphere.
    - $E_T = E_k + E_p \longrightarrow E_T = \frac{1}{2}mv^2 - \frac{GMm}{r}$.
        - The mass, M, of the sphere is given by: $M = \frac{4}{3}\pi r^3 \rho$
    - $E_T = \frac{1}{2}m(H_0 r)^2 - \frac{G(\frac{4}{3}\pi r^3 \rho)m}{r}$
- The galaxy will continue moving as long as it has enough kinetic energy.
- At the limit $E_T = 0$ (where the GPE will cancel out the E_k), the formula is:
    - $\frac{1}{2}m(H_0 r)^2 = \frac{G(\frac{4}{3}\pi r^3 \rho_c)m}{r}$

        - ρ_c is the critical mass.
    - ρ_c will be given by: $\rho_c = \frac{3H_0^2}{8\pi G}$

        - Note: Since the equation is only dependent on constants, the value of ρ_c is solely dependent on the accuracy of H_0.
    - This gives the critical density of the universe, which if its equal, the universe will no longer expand.

- The cosmic scale factor and time
    - The ratio of the actual density of matter in the universe (denoted by "ρ") to the critical density is termed the density parameter.
        - $\Omega_0 = \frac{\rho}{\rho_c}$
    - There are three possibilities for the fate of the universe:
        - If $\Omega_0 = 1$ (or $\rho = \rho_c$), the universe will remain flat and expand to a maximum value.
            - The rate of expansion would decrease over time.
            - This is thought to be the most likely option.
        - If $\Omega_0 < 1$ (or $\rho < \rho_c$), the universe will be open and would continue to expand forever.
        - If $\Omega_0 > 1$ (or $\rho > \rho_c$), the universe would be closed.
            - It will eventually stop expanding, and collapse ending with a "big crunch".
    - The accelerating expansion of the universe might be explained by the presence of dark energy.
        - The cosmic scale factor (R) is essentially the relative size, or "radius", of the universe.
    - Each model gives an Ω_0 value that is based on the total matter in the universe.
        - The explanation for the accelerated universe depends on the concept of the hypothetical dark energy which outweighs the gravitational effects of baryonic and dark matter.

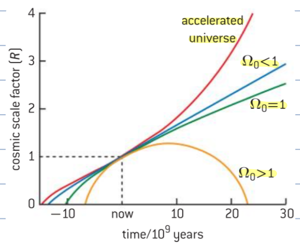
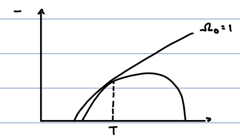


Figure 4 Variation of R for different density parameters.

- The cosmic scale factor & temperature

- the wavelengths of the radiation emitted by a galaxy will always be in line with the cosmic scale factor (R).
    - As the space (universe as a whole) expands, the wavelength will also increase.
- the wavelength and the cosmic scale factor are inversely proportional to the temperature.
    - this is because as the universe expands, the temperature will decrease. Think of how the temperature varies with an increase in volume of the container.
    - $T \propto \frac{1}{R}$ or $T \propto \frac{1}{\lambda}$

- Worked example
    - the closed universe means that Ω_0 will be greater than 1, $\rho > \rho_c$.
        - this will mean that the universe will stop expanding and instead collapse under gravity.
    -



- the acceleration of the universe will have to be taken into consideration to see and calculate the time for the age of the universe
- i) Redshift from a Type Ia supernova
  ii) Dark Energy outweighing the gravitational force of Dark Matter and baryonic.

- Evidence of Dark Matter
    - Star of mass m near the center of a spiral galaxy of total mass M.
        - the average density in the region is ρ.
        - the star moves in a circular orbit with an orbital velocity v and radius r.
        - By equating Newton's law of gravitation to the centripetal force we obtain:
            - $\frac{GmM}{r^2} = \frac{mv^2}{r}$
              $\frac{GM}{r} = v^2$
        - In terms of the density and taking the central hub to be spherical, this gives:
            - $\frac{G\frac{4}{3}\pi r^3\rho}{r} = v^2 \longrightarrow v = \sqrt{\frac{G4\pi\rho}{3}}$ or $v = \text{constant} \cdot r$
            - Velocity is directly proportional to the radius.
    - If the star were in a less densely populated area of the galaxy, it would be expected that the star would orbit like planets orbit the sun.
        - the galaxy would behave as if the total mass were concentrated at its center.
        - the star would move with nothing to impede its orbit.
            - Giving: $\frac{GM}{r} = v^2$, so $v = \frac{1}{\sqrt{r}}$.
    - therefore, when the star is plotted against the distance from the center of the galaxy, it's expected to see a rapidly increasing linear section that changes to a decaying line at the edge of the hub.
        - this would be done by measuring the speed from the redshifts of the rotating stars.
        - By doing this, it can be seen that the speed of the stars, far out into the regions beyond the arms of the galaxy, are moving at approximately the same speed as those in the galaxy.
            - the explanation for this is the presence of dark matter.
                - the dark matter is forming a halo around the outer rim of the galaxy, causing this effect.
                - the matter isn't normal nor is it baryonic, and emits no radiation.
    -

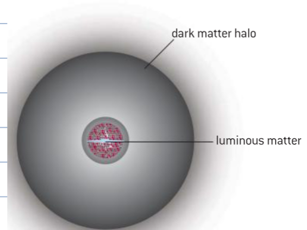


Figure 6 Dark matter halo surrounding a galaxy.

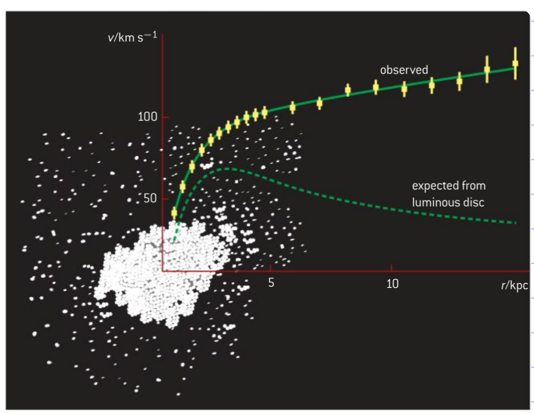


Figure 7 The rotation curve for the spiral galaxy M33.

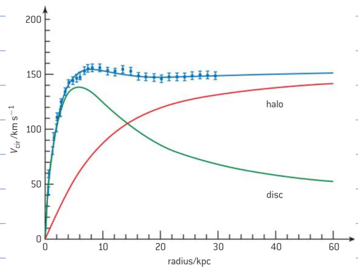
- The following figure shows that the experimental curve (labelled "halo") can be determined by assuming that the halo adds sufficient mass to that of the galactic disk.
  - This maintains a high rotational speed well away from the galactic centre.
  - 

Figure 8 distribution of dark matter in NGC 3198.

- Other evidence of dark matter is:
  - the velocities of galaxies orbiting each other in clusters.
    - the galaxies are emitting less light than they should in relation to the amount of mass suggested by their velocities.
  - the gravitational lensing effect of radiation from distant objects (e.g. quasars).
    - Gravitational lensing effect is when a large amount of matter, like clusters of galaxies, will bend / distort the light by their gravitational pull.
      - the more massive the object, the more lensing occurs.
    - A quasar is an extremely luminous active galactic nucleus, in which a supermassive black hole (mass from millions to billions $M_\odot$) is surrounded by an accreting disk.
    - the radiation is distorted more than expected.
  - the X-ray images of elliptical galaxies show the presence of haloes of hot gas extending well outside the galaxy.
    - For this gas to be bound to the galaxy, the mass of the galaxy has to be 90% greater.
      - Likely due to dark matter.
- Some candidates for the nature of dark matter are:
  - MACHOs.
    - MAssive Compact Halo Objects.
    - these include black holes, neutron stars, and small stars such as brown dwarfs.
      - These are high density (compact) stars at the end of their lives and might be hidden because they're not near any luminous objects.
      - Detected by gravitational lensing.
    - It's questionable whether or not there are sufficient MACHOs to provide all the dark matter.
  - WIMPs
    - Weakly Interacting Massive Particles.
      - These are subatomic that aren't made of ordinary matter (non-baryonic).
      - they are weakly interacting because they pass through ordinary, baryonic, matter with little effect.
      - Massive means they have mass, although very small.
      - Unimaginably large quantities of WIMPs would be required to produce the amount of dark matter.
      - Neutrinos could be an explanation.
        - Apart from this, the theory for WIMPs depends on theoretical particles called axions and neutralinos.

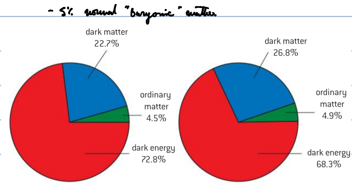
- Dark Energy
  - Dark energy is what is considered to be expanding the universe at an accelerating rate.
  - The data suggests that dark energy is 68% of the universe.
    - 27% dark matter.
    - 5% normal "baryonic" matter.
  - 

Figure 9 The mass/energy recipe for the universe.

- It's suggested that dark energy is a property of space, and as space expands so does the amount of dark energy.
  - This form of energy would cause the universe to expand at an accelerating rate.

- Anisotropies in the CMB

  - Even though the CMB is isotropic (same values even when measured from different directions), there are minute temperature fluctuations called anisotropies.
    - Low level temperature fluctuations were detected.
    - These variations in temperature are thought to arise due to tiny, random variations in density, implemented during cosmic inflation.
      - Cosmic inflation is the period of accelerated expansion that occurred immediately after the Big Bang.
  - The pattern shown in variation demonstrates the differences present on release of the radiation (when universe = 380,000 years old).
    - Fluctuations that would later grow into galaxies and galaxy clusters under gravity.
  - There is a cold spot extending over a patch of sky.
    - The standard model doesn't take into account all of the variations, and expects the universe to be isotropic.
  - The Planck data identifys Hubble's constant at 67.15 $km\,s^{-1}\,Mpc^{-1}$ (less than the 70 $km\,s^{-1}\,Mpc^{-1}$).